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IS PROMETHEUS REALLY BOUND?

Introduction

Although the capture concept seems to be important to packet radio, it proves difficult and slippery to trap in a succinct model. Roberts, Klienrock, Leung, Abramson, and Gaarder have all shown that the throughput of a packet radio channel can be increased if the strongest of two interfering signals always succeeds.

In these models it is assumed that capture is a binary event--whenever the stronger signal is received with enough extra power it is assumed to successfully communicate. A parameter, the "capture-ratio" (C.R.) is defined to measure the relative strength of signals at which capture will occur.

The degree of improvement in throughput attributable to capture depends strongly upon the capture ratio. Abramson shows that perfect capture can improve performance by a factor of e. No one has obtained precise results for imperfect capture (capture ratios greater than 0 dB), but Roberts' and Leung's results suggest that very little improvement is available when capture ratios exeeed about 10 dB. To estimate throughput achievable with different access techniques, we require some estimate of the capture ratio. The purpose of this note is to provide some insight into the capture ratio we might expect by examining the capture probability for PSK signals.

We first define two equivalent measures of the capture ratio and then observe that in the absence of noise for ideal receivers of digital signals the capture ratio is precisely 0 dB. That is, the stronger signal will always successfully communicate. When noise is present the capture event is probabilistic and the probability that a stronger signal will be successful depends upon the signal to noise ratio as well as the duration of interference.

We explore the capture phenomenon by defining a hypothetical packet radio network and placing a wandering observer, called Argus in the net. Some insight into capture is provided by curves of the probability of capture which Argus observes.

Summary

We can summarize our results with the following statements:

1. The noise-free capture ratio is 0 dB for digital systems.
2. There is no such thing as a capture ratio in the presence of noise.
3. An Aloha mode will benefit more from capture then a Slotted Aloha mode.
4. If there was a capture ratio in noise it would be in the range 1- to 8-dB for Aloha and 2- to 14-dB for Slotted Aloha systems using CPSK.

Capture Ratio Definition

Roberts (A.S.S. Note No. 8) defines a capture ratio, C.R. in terms of the minimum value of the ratio of power in the stronger to power in the weaker signal at which the stronger signal will capture the channel.

$$CR = 10 \log \left(\frac{P_{strong}}{P_{weak}} \right)_{\text{at capture}}$$

Abramson (P.R. Note No. 49) defines a parameter, a, as the minimum value of the ratio of weak-signal range to strong-signal range at which the strong signal will capture the channel

$$a = \left(\frac{R_{weak}}{R_{strong}} \right)_{\text{capture}}$$

These can be related using a model of propagation loss vs. range. If received power varies as R^{-b}, then

$$CR = 10b \log (a)$$

Both Roberts and Abramson have assumed a capture ratio independent of range. Since the received power varies with range, this is an implicit assumption that the capture ratio is independent of received power. When noise is ignored, this is a good assumption, and it proves very useful in providing an upper bound on channel throughput. When we wish to include the effects of noise, we must drop the assumption. Let us consider both cases.

Noiseless Capture

We think of capture in terms of two signals occurring simultaneously; in this context three events may occur: the stronger signal may be successful, the weaker signal may be successful, or neither signal may be successful. The first of these events is the capture event.

In the absence of noise the stronger signal will always capture the channel for digital angle-modulated signals and ideal receivers. Consider, for example, PSK signals. The decision variable will always have the sign of the stronger of the two signals, because only linear processing preceeds the decision. In the case of DPSK signals the product of reference bit and current bit will always have the sign associated with the stronger signal. For FSK signals noncoherently detected, the energy will always be greater at the output of the matched filter containing the stronger signal. Hence, in the absence of noise, the capture ratio will be 0 dB, or $a = 1$.

Noise and Capture

When noise is present some packets will be successful and others will fail even when there is no interference. Interference acts to decrease the success rate. Since capture is roughly equatable to successful packet transmission, capture is a probabilistic event, and the capture ratio can only be understood in terms of the probability of success for a packet.

The probability of success for a packet, called the capture probability, P_c, will depend upon several factors. These include the signal-to-noise ratio, the signal-to-interference ratio, and the degree of overlap between the signal packet and the interfering packet. To explore the effect of these parameters, suppose that we define a hypothetical network of our usual unperturbable Poisson sources communicating with a central receiver using coherent PSK transmitters and operating in an Aloha mode. Suppose that the sources are uniformly distributed over a flat earth, and that the propagation loss from a source to the central point obeys an inverse-square power law. Finally, suppose that each source has a transmitter with fixed power so that the signal-to-noise ratio at the receiver depends upon the inverse-second power of the range.

Now introduce an observer into this network and allow him to wander from source-to-source and determine the capture probability at each source. Call this observer Argus, so that Sisyphus will not be lonely.

According to the results of PRN Nos. 49 and 60, Argus will observe some range beyond which no one can communicate. Because the network is efficient in power use, it will be adjusted so that this distance coincides with the maximum range for reliable communication. For PSK signals, the signal-to-noise ratio will be about 9 dB for sources at the Sisyphus distance. Because of the uniform distribution of sources, and the inverse-square power propagation law Argus will notice that 50 percent of the sources communicate with signal-to-noise ratios between 9- and 12-dB, and 75 percent use signal-to-noise ratios between 9- and 15-dB.

Argus will also notice that when two sources interfere, because of the Poisson distribution of start-times 85 percent of the interferences will exceed 100 bits out of a 1000 bit packet when the network operates in an Aloha mode.* Of course all interfering packets will interfere for

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* Let probability of no interference = P_s, then the probability that packets will overlap by a fraction f given that they overlap is just $1-\frac{(1-P_s)^f}{(1-P_s)}$. For Aloha at maximum throughput $P_s = \frac{1}{e}$, and $1-\frac{(1-\frac{1}{e})^{1/10}}{1-\frac{1}{e}} = .85$.

1000 bits when slotted Aloha is used.

Now if Argus observes the probability of capture by counting successes at the 84 percent, 68 percent, and 37 percent population contours he will find a variation depending upon the relative strength of the interfering signal. This variation is shown in Figure 1a for Aloha, and Figure 1b for Slotted Aloha operation.*

Discussion

Consider the Aloha case shown in Figure 1a. When Argus is inside the 12 dB contour (for about 50 percent of the sources) he will observe that most packets are successful unless an interfering source occurs with a strength that is more than about 7- or 8-dB below the observed source. Thus, we might say that capture probability is "high" for signal/interference ratios exceeding about 8 dB. Similarly, capture probability is "low" for signal/interference ratios less than about 1 dB. Thus, the noisy capture phenomenon might improve the Aloha system performance about as much as a "noiseless" capture ratio somewhere between 1- and 8-dB. Just how much improvement is available might be estimated using the noiseless model, but a more accurate answer would require a very complex model and is probably not worth the trouble.

The situation is somewhat worse for slotted Aloha as shown in Figure 1b. Using arguments as above, the capture ratio may be anywhere from about 2- to 14-dB. This means that capture will not be as helpful for slotted Aloha as for Aloha.

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* See appendix for derivation of curves.

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Appendix

Performance of CPSK Signals in Interference

For this note we have examined the case of a binary phase-shift keying (PSK) modulation scheme. Such a scheme is a likely candidate for packet radio since it is easily implemented and one of the most efficient in terms of both power and bandwidth utilization. Such a scheme is commonly used to generate spread-spectrum signals, so our analysis may prove useful when considering these signals as well; however, a processing gain factor will enter into those calculations which we will leave to a later time. Our results can be suitably interpreted to apply to several other modulation schemes including quad-phase-shift-keying (QPSK) and minimum-shift keying (MSK). The curves presented show qualitatively the effect of interference to be expected in almost any digital modulation scheme in which information is conveyed by quantized changes in the phase of a carrier.

A Phase-Shift Keying Model

For this first look we have used the simplest possible model that seemed to retain the necessary features of a realistic radio channel, except that we have ignored the effects of multipath.

In a PSK signal, information is conveyed by the phase of the carrier. In a binary PSK signal the information is used to select either a 0° or a 180° phase shift of the carrier. Thus a transmitted signal will be

$$s(t) = m\sqrt{2P}\cos(wt) \qquad 0 \le t \le T$$

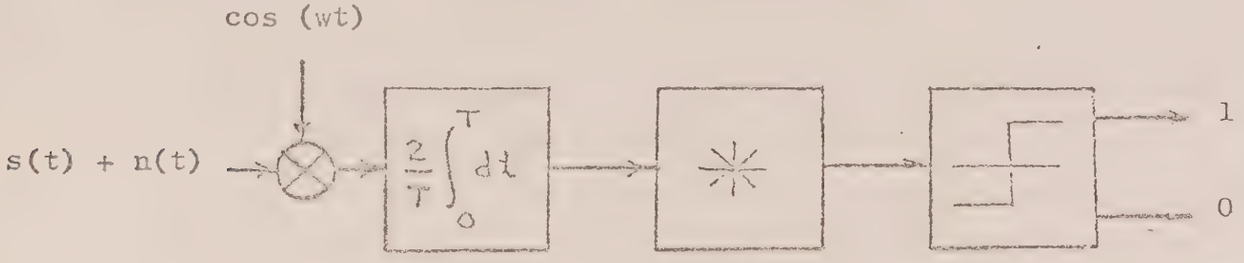
where $m = +1$ to send a "one"

$= -1$ to send a "zero"

T = bit duration

P = peak power = (average power over bit duration)

An ideal receiver for such a signal in white Gaussian noise consists of a synchronous correlator as sketched below.



If the received signal is $s(t) + n(t)$ where the noise is white with spectral density $N_o/2$ and Gaussian, then the correlator output at $t = T$ will be

$$z = m\sqrt{2P_r} + \eta$$

P_r = peak received power = transmitted power attenuated by channel

where η is a Gaussian random variable of zero mean and variance

$$\frac{N_o}{T}$$

Z is the decision variable. The optimum receiver makes a decision based solely on the sign of Z. If $Z > 0$ the receiver decides a "one" was sent, otherwise a "zero." The receiver makes an error whenever the noise is such that the wrong decision is made; e.g., when $m = +1$ an error is made when $\eta < -\sqrt{2P}$.

Thus the probability of an error is simply

$$P_e = \frac{1}{\sqrt{2\pi \frac{N_o}{T}}} \int_{\infty}^{-\sqrt{2P_r}} e^{\frac{-x^2 T}{2N_o}} dx = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\gamma}\right)$$

where $\gamma = \frac{P_r T}{N_o}$ and $\text{erfc} = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$

Note that the energy received per bit is simply $P_r T$, so that the performance depends only on the ratio of energy received per bit to background noise level (γ). For a fixed channel noise and attenuation, and a fixed peak power transmitted, the performance can be adjusted only by varying the bit duration (or signal bandwidth).

Since the source of error is assumed to be white Gaussian noise, errors will be independent, and the packet error probability, P_m, for an m-bit packet is

$$P_m = 1 - (1-P_e)^m$$

Now suppose that we include a second interfering signal at the receiver input. Assume that the interfering signal has a random bit-start time, and random phase relative to the desired signal but has the same frequency and bit-rate. The interfering signal, at the receiver may be modeled as

$$s_I(t) = \alpha_1 \sqrt{2P_I} \cos(wt + \theta) \quad 0 \le t \le \tau$$

$$\alpha_2 \sqrt{2P_I} \cos(wt + \theta) \quad \tau \le t \le T$$

where $\alpha_1 = 1$ if the previous bit was a one
-1 if the previous bit was a zero
$\alpha_2 = 1$ if the present bit was a one
-1 if the present bit was a zero
θ is a random variable uniform $\varepsilon(0, 2N)$
τ is a random variable uniform $\varepsilon(0, T)$

In this case the decision variable, z is

$$z = m\sqrt{2P_r} + \alpha_1 \frac{\tau}{T} + \alpha_2 \left(\frac{T-\tau}{T}\right) \sqrt{2P_I} \cos\theta + \eta$$

Thus the effect of the interfering signal is simply to change the effective signal-to-noise ratio on a bit-by-bit basis. The performance of the system in the presence of interference can be found in principle from the previous relation by averaging over α_1, α_2, τ, and θ:

$$P_e = \int \frac{1}{2} \operatorname{erfc} \left\{ \left[\frac{P_r T}{N_o} + \left(\alpha_1 \frac{\tau}{T} + \alpha_2 \left(\frac{T-\tau}{T}\right) \right) \frac{P_I}{N_o} T \cos \theta \right]^{1/2} \right\} dP(\alpha_1, \alpha_2, \tau, \theta)$$

The relationship can be modified for easier numerical calculation but no additional insight is provided and the manipulations are too detailed for this note. Instead we provide curves of the relationship between P_e, $\frac{P_r T}{N_o}$, and $\frac{P_I}{P_r}$ in Figure 2 and 3. These curves were calculated using a digital computer to perform the required numerical integration. If they are compared to curves of CPSK performance in the presence of independent interference one will find that performance of the system with this type of dependent interference is better.

To convert these curves to the curves of Figure 1 we must compute the probability of a message error from the above results.

Let

$$P_e(\alpha_1, \alpha_2, \tau, \theta) = \frac{1}{2} \operatorname{erfc} \left\{ \left[\frac{P_r T}{N_o} + \left(\alpha_1 \frac{\tau}{T} + \alpha_2 \frac{(T-\tau)}{T} \right) \frac{P_I T}{N_o} \cos \theta \right]^{\frac{1}{2}} \right\}$$

and

$$P_e(\tau, \theta) = E_{\alpha_1, \alpha_2} \left[P_e(\alpha_1, \alpha_2, \tau, \theta) \right]$$

Then because the noise is white, the bit error probability conditioned on τ, θ will be independent from bit-to-bit and

$$P \text{ (message error/L bit overlay)} = P_m (L)$$

$$P_m (L) = E_{\tau,\theta} \left(1 - \left[1 - P_e (\tau, \theta) \right]^L \right)$$

For small L $P_e (\tau, \theta)$ we have

$$P_m(L) \simeq L \, E_{\tau,\theta} \left[P_e(\tau, \theta) \right] = L \, P_e$$

Thus if we scale P_e by a factor of L we can interpret the curves of Figure 2 and 3 as message error rate curves below $P_m = 10^{-1}$ or so.

Now, the probability of success, or capture, P_c is related to the message error probability as

$$P_c = 1 - P_m,$$

therefore, the curves of Figure 1 can be derived from Figure 3 by scaling P_e by L, subtracting from 1, and plotting with P_c on a linear scale.

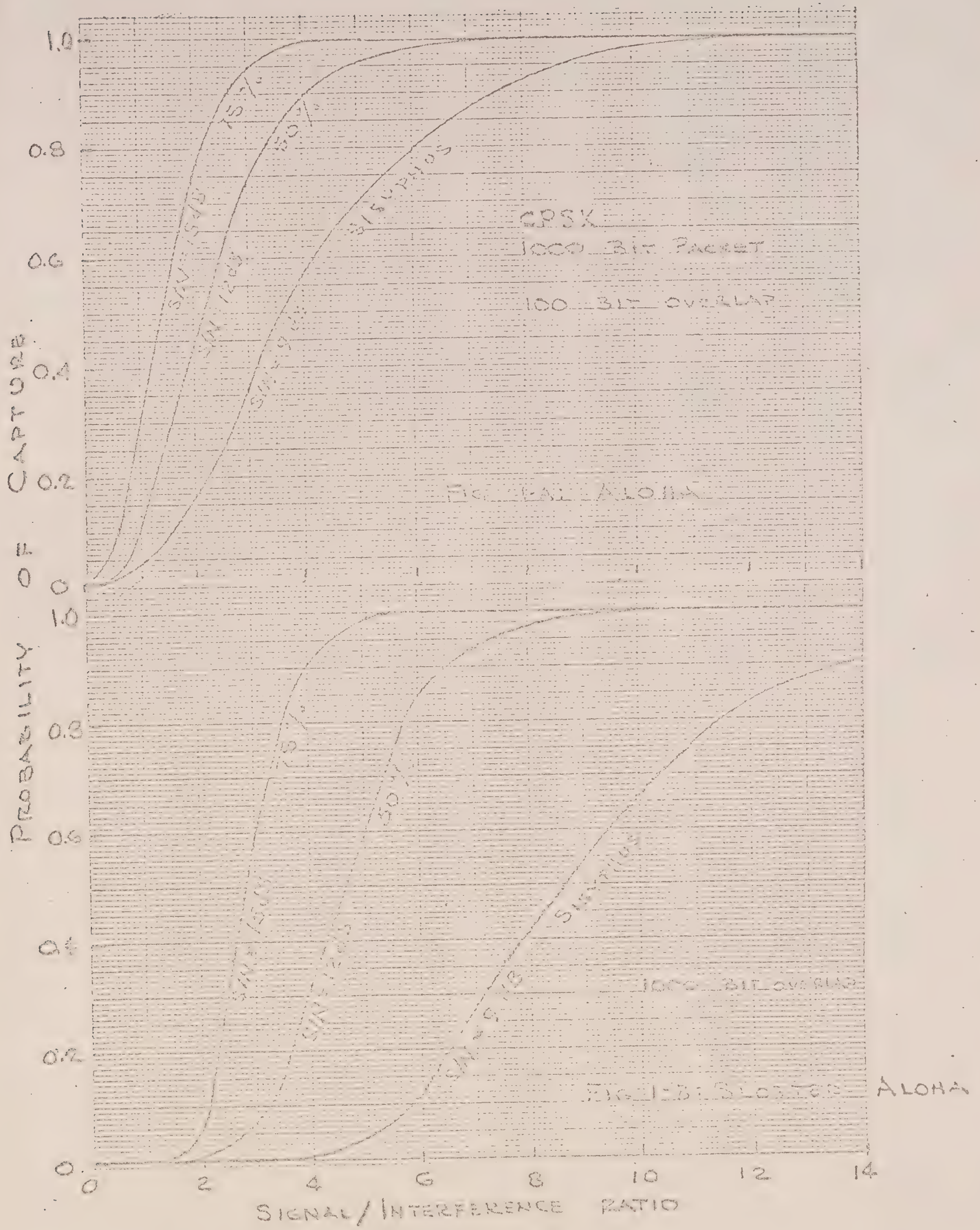


FIG 1 - CAPTURE PROBABILITY

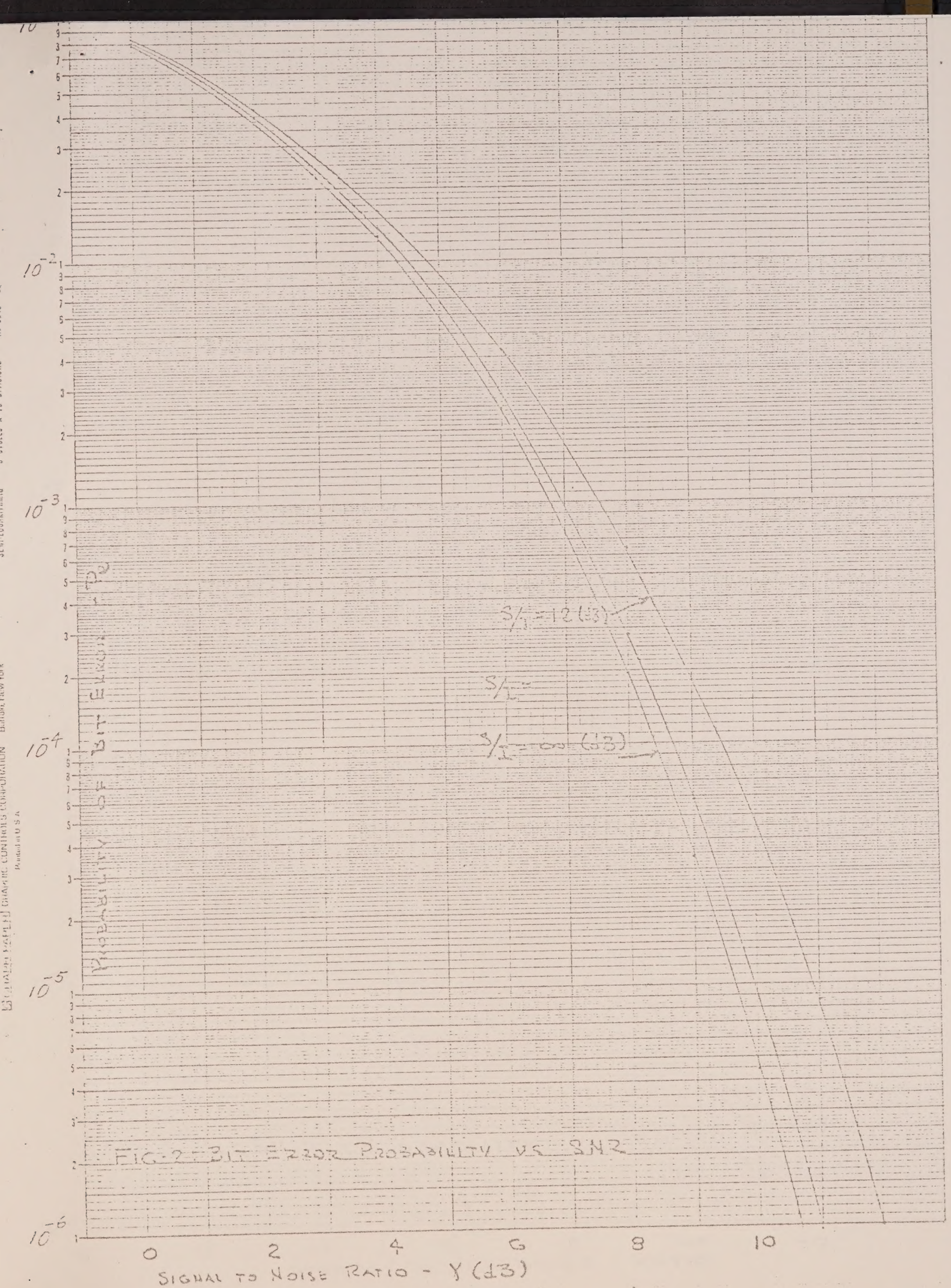


FIG. 2 - BIT ERROR PROBABILITY VS SNR

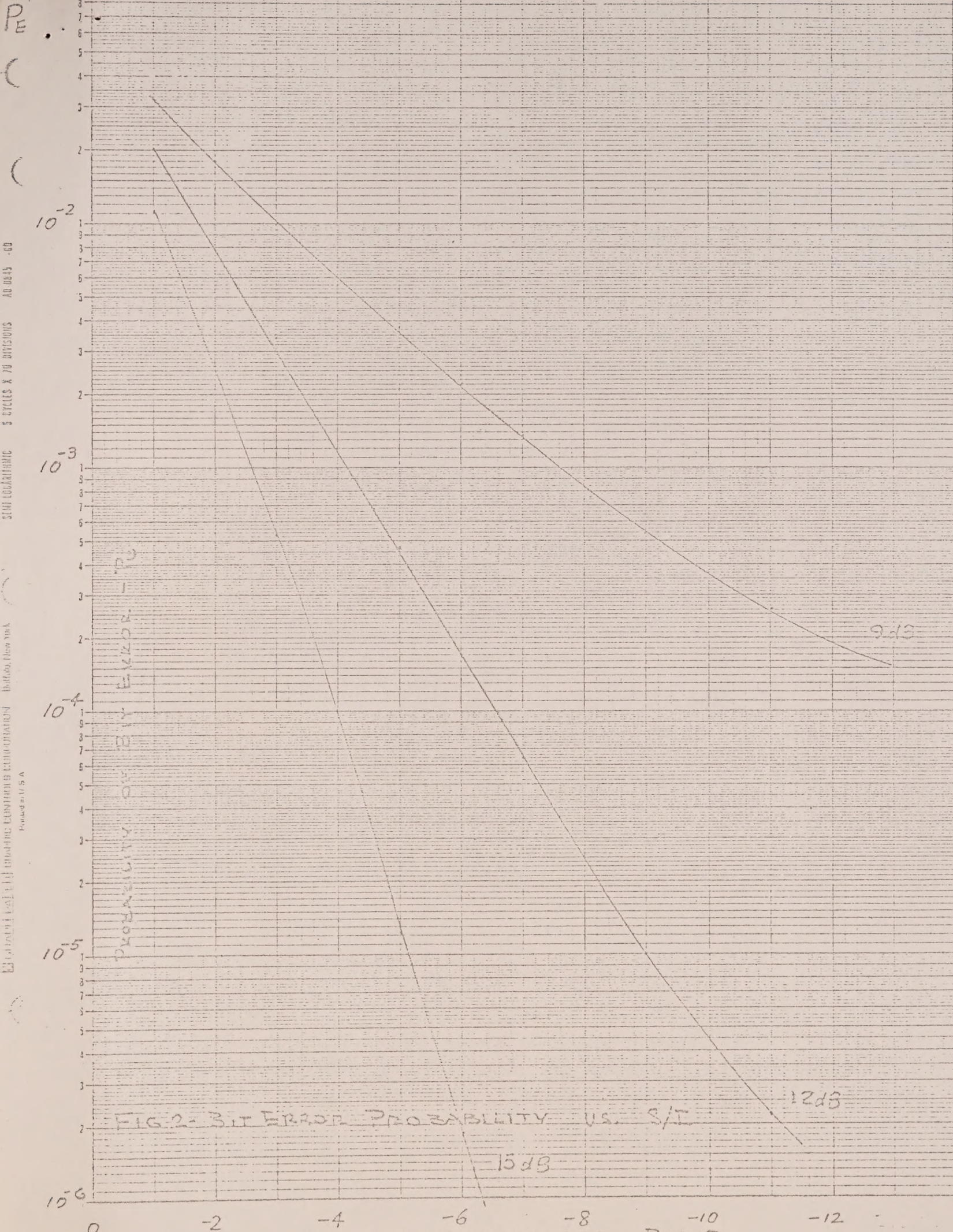


FIG 2 - BIT ERROR PROBABILITY VS. S/I